Exercise 1

We consider a half cylinder of radius a placed in an incompressible fluid of density ρ. The fluid domain is bounded by an infinite plate above which sits a half cylinder. Far from this obstacle, the flow is uniform and characterised by a velocity field $\mathbf{v} = U_\infty \mathbf{e}_x$ under uniform pressure p_∞, as seen on the figure.

On the surface of the half cylinder, a little ventilation hole has been drilled in $\theta = \theta_a$ where the mechanical equilibrium is imposed between the air in the interior of the half-cylinder and the fluid at the location of the orifice.

The flow is assumed to be potential. The flow velocity in polar co-ordinates can be expressed in terms of streamfunction as $u_\theta = \frac{\partial \psi}{\partial r}$ and $u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$. The velocity potential and streamfunction for a uniform flow and doublet in polar co-ordinates can be expressed as

Uniform Flow: $\phi = Vrcos\theta$ and $\psi = Vrsin\theta$

Doublet: $\phi = \frac{Krcos\theta}{R}$ and $\psi = \frac{-Ksin\theta}{r}$

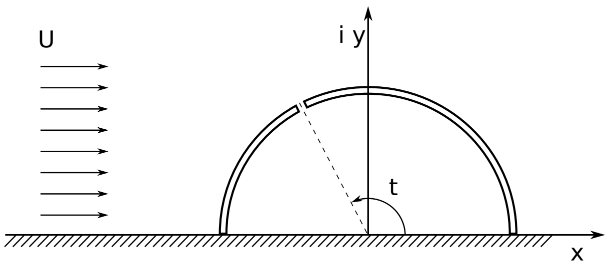

Figure 1. *Hemi-cylindrical tent sitting on a plane ground in a uniform flow. The tent displays a ventilation orifice located at the angle $\theta = \theta_a$.*

1. Using the above information, and imposing the boundary condition express the streamfucntion for the flow around the cylinder.

2. What is the pressure on the exterior surface of the cylinder, i.e. $p_e(\theta) = p(r = a, \theta)$.

3. Determine the interior pressure p_i and calculate the total force applied on the tent. Deduce that there exists at least one value of θ_a for which the force on the tent cancels out.

Exercise 2 : potential flow around a slightly distorted circle

The objective of this exercise is to determine an approximate acyclic potential flow solution for the uniform flow around an obstacle slightly departing from the circular shape $r = a(1 - \epsilon \sin^2(\theta))$.

a) Plot the shape of the obstacle.

b) Recall the acyclic stream function associated to the flow around an undistorted circle of radius a with uniform flow $U_\infty \mathbf{e}_x$ at infinity.

c) We are looking for a complex potential under the form

$$\psi = \psi_0 + \epsilon\psi_1 + O(\epsilon^2) \tag{1}$$

Express ψ_0 in polar coordinates (r, θ) using .

d) Show that, using the flattening of the boundary condition on the distorted cylinder and neglecting higher order terms in ϵ, ψ_1 is solution of

$$\Delta\psi_1 = 0 \tag{2}$$

with boundary conditions

$$\psi_1(\infty) = 0; \psi_1(a, \theta) = a \sin^2(\theta) \frac{\partial \psi_0}{\partial r}(a, \theta) \tag{3}$$

e) Show that this last boundary condition can be reexpressed as

$$\psi_1(a, \theta) = aU_\infty/2(3\sin(\theta) - \sin(3\theta)) \tag{4}$$

where we have used

$$\sin^3(\theta) = (3\sin(\theta) - \sin(3\theta))/4 \tag{5}$$

f) Using the expression of the scalar Laplacian in polar coordinates, show that

$$\psi_1 = \frac{3a^2U_\infty}{2r}\sin(\theta) - \frac{a^4U_\infty}{2r^3}\sin(3\theta) \tag{6}$$

is solution.